## Kakutani's interval splitting scheme

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Put a random – i.e. uniformly distributed – point $X_1$ in the unit interval (0.1). Choose the longest of the resulting two subintervals (0, $X_1$) and (X1,1) and put a random point $X_2$ in this interval. Continue in this way, choosing $X_k$ randomly in the longest of the k intervals into which $X_1, X_2, \ldots, X_{k-1}$ subdivide (0,1). Kakutani asked whether in the long run the points become evenly – i.e. uniformly - distributed in (0,1). There are obvious reasons why this should be true, but the proof turned out to be a different matter altogether.

Once we have a proof, we can resolve some related matters. For instance, one can ask how the speed of convergence compares with well-studied classical case where the random variables $X_1, X_2, \ldots$ are independent and uniformly distributed on (0,1). While answering this question we come across some interesting and unexpected phenomena. All of this is based on joint work with Ronald Pyke.