



The University of Chicago  
Department of Statistics  
Seminar Series

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**TILMANN GNEITING**

Department of Statistics  
University of Washington

**Mean, Median, Mode, More**

**MONDAY, March 9, 2009 at 4:00 PM**  
**133 Eckhart Hall, 5734 S. University Avenue**

*Refreshments following the seminar in Eckhart 110.*

**ABSTRACT**

Suppose that we are to predict a future quantity of interest. Our favorite statistical technique provides a predictive distribution, say  $F$ . However, as it turns out, we are required to issue a single-valued point forecast. How are we going to proceed? Evidently, our strategy will depend on the loss structure. If the loss function is quadratic, the mean of  $F$  is the unique optimal point predictor. Under symmetric piecewise linear loss, we pick the median of  $F$ , under zero-one loss its mode.

Are standard loss functions realistic? Typically, no. Are standard predictors, such as quantiles, compatible with realistic loss structures? Perhaps surprisingly, typically, yes. Indeed, quantiles arise as optimal point predictors under a general class of economically relevant loss functions, to which we refer as generalized piecewise linear (GPL). The level of the quantile depends on a generic asymmetry parameter that reflects the possibly distinct costs of under-prediction and over-prediction. A loss function for which quantiles are optimal point predictors is necessarily GPL, similarly to the classical fact that a loss function for which the mean is optimal is necessarily a Bregman function.

The practical relevance of the choices in the transition from the predictive distribution to the point forecast will be illustrated on the Bank of England's density forecasts of United Kingdom inflation rates, and probabilistic predictions of wind energy resources in the Pacific Northwest.