The University of Chicago

Department of Statistics

Ph.D. Seminar

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"The Problem of Coexistence in Multi-type Competition Models"

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ABSTRACT

Two species compete for territory along their boundaries. Each site of the space may contain only one representative (also referred to as particle) of either type. The spread mechanism for both species is the same: each particle produces offspring independently of other particles and can place them only at the neighboring sites that are either unoccupied, or occupied by particles of the opposite type. In the second case the old particle is killed by the newborn. The rate of birth for each particle is $\lambda_\tau \times$ (# of neighboring sites available for expansion), for $\tau = 1, 2$ corresponding to the types 1 and 2. The main problem we address concerns the possibility of the long-term coexistence of the two species. We consider the model on two spaces: the integer lattice $\boldsymbol{Z}^d$ and the homogeneous tree $\mathcal{T}_\lceil$ where each vertex has exactly $d + 1$ neighbors. For both spaces, we have shown that if we start the process with finitely many representatives of each type, then, assuming $\lambda_1 = 1$, there exists a critical rate $\lambda_{co} > 1$ such that, for all $\lambda_2 \in (1, \lambda_{co})$, there is a positive probability of coexistence. For all $\lambda_2 \in (\lambda_{co}, \infty)$ only one type of particle can survive.

In order to analyze the competition model on $\mathcal{T}_\lceil$ we have also considered the Richardson model and the biased voter model on a homogeneous tree. The analogs of the Shape theorem for the Richardson model and the biased voter model have been proved.
