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DISSERTATION PRESENTATION AND DEFENSE

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Some Metric Properties of Planar Gaussian Free Field

FRIDAY, June 9, 2017, at 2:30 pm
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ABSTRACT

We will discuss the properties of some metrics arising from two-dimensional Gaussian free field (GFF), namely the Liouville first-passage percolation (Liouville FPP), the Liouville graph distance and an effective resistance metric. Roughly speaking, Liouville FPP is the shortest path metric in a planar domain D where the length of a path P is given by $\int_P e^{\gamma h(z)}|dz|$ where h is the GFF on D and $\gamma > 0$. We will present an upper bound on the expected Liouville FPP distance between two typical points for small values of γ (the *near-Euclidean* regime). A similar upper bound can be given for the Liouville graph distance which is, roughly, the minimal number of Euclidean balls with comparable Liouville quantum gravity (LQG) measure whose union contains a continuous path between two endpoints. Our bounds seem to be in disagreement with Watabiki's prediction (1993) on the random metric of Liouville quantum gravity in this regime. Next we will discuss some asymptotic estimates for effective resistances on a random network which is defined as follows. Given any $\gamma > 0$ and for $\eta = \{\eta_v\}_{v\in\mathbb{Z}^2}$ denoting a sample of the two-dimensional discrete Gaussian free field on $\mathbb{Z}^2$ pinned at the origin, we equip the edge (u, v) with conductance $e^{\gamma(\eta_u+\eta_v)}$. The metric structure of effective resistance plays a crucial role in our derivation of the estimates. The primary motivation behind this metric is to understand the random walk on $\mathbb{Z}^2$ where the edge (u, v) has weight $e^{\gamma(\eta_u+\eta_v)}$. This random walk was studied in the physics literature as a model for diffusion in a random potential with log-correlations. Using our estimates on effective resistances, we can show that for almost every η, this random walk is recurrent and that, with probability tending to 1 as $T \to \infty$, the return probability at time $2T$ decays as $T^{-1+o(1)}$. In addition, we can prove a version of subdiffusive behavior by showing that the expected exit time from a ball of radius N scales as $N^{\psi(\gamma)+o(1)}$ with $\psi(\gamma) > 2$ for all $\gamma > 0$. The content of this talk is based on several papers co-authored with Jian Ding and Marek Biscup.
